Άσκηση (για επίλυση)

Απάντηση:

$$\lim_{n\to+\infty}(\alpha_n) = 1$$

Άσκηση

$$\lim_{n\to+\infty}(\alpha_n) = \lim_{n\to+\infty}\left(\frac{n^2-1}{n^2+1}\right)^{n^2} = \lim_{n\to+\infty}\left(\frac{\frac{n^2}{n^2}-\frac{1}{n^2}}{\frac{n^2}{n^2}+\frac{1}{n^2}}\right)^{n^2} \Rightarrow$$

$$\Rightarrow \lim_{n\to+\infty}(\alpha_n) = \lim_{n\to+\infty}\left(\frac{1-\frac{1}{n^2}}{1+\frac{1}{n^2}}\right)^{n^2} = \frac{\lim_{n\to+\infty}\left(1-\frac{1}{n^2}\right)^{n^2}}{\lim_{n\to+\infty}\left(1+\frac{1}{n^2}\right)^{n^2}} \Rightarrow$$

$$\Rightarrow \lim_{n\to+\infty}(\alpha_n) = \frac{\left[\lim_{n\to+\infty}\left(1+\frac{1}{(-n^2)}\right)^{n^2}\right]^{-1}}{\lim_{n\to+\infty}\left(1+\frac{1}{n^2}\right)^{n^2}} \Rightarrow$$

$$\Rightarrow \lim_{n\to+\infty}(\alpha_n) = \frac{e^{-1}}{e} \Rightarrow \boxed{\lim_{n\to+\infty}(\alpha_n) = \frac{1}{e^2}}$$

$\lim_{n\to+\infty}\left(1+\frac{1}{n^2}\right)^{n^2} = e$, αφού $n^2 > 0$ και $\lim_{n\to+\infty} n^2 = +\infty$

$\lim_{n\to+\infty}\left(1+\frac{1}{(-n^2)}\right)^{-n^2} = e$, αφού (- n^2) < 0 και $\lim_{n\to+\infty}(-n^2) = -\infty$.

$$\lim_{n\to+\infty}(\beta_n) = \lim_{n\to+\infty}\left(\frac{2n+3}{2n}\right)^{3n+2} = \lim_{n\to+\infty}\left(1+\frac{3}{2n}\right)^{3n+2} \Rightarrow$$

$$\Rightarrow \lim_{n\to+\infty}(\beta_n) = \lim_{n\to+\infty}\left(1+\frac{3}{2n}\right)^{3n} \lim_{n\to+\infty}\left(1+\frac{3}{2n}\right)^{2} \Rightarrow$$

$$\left.\begin{array}{c}\Rightarrow \lim_{n\to+\infty}(\beta_n) = \left[\lim_{n\to+\infty}\left(1+\frac{1}{\frac{2n}{3}}\right)^{\frac{2n}{3}}\right]^{\frac{9}{2}} \lim_{n\to+\infty}\left(1+\frac{3}{2n}\right)^{2} \\ \Rightarrow \lim_{n\to+\infty}(\frac{3}{2n}) = 0 \\ \left[\lim_{n\to+\infty}\left(1+\frac{1}{\frac{2n}{3}}\right)^{\frac{2n}{3}}\right] = e, \text{αφού} \frac{2n}{3} > 0 \text{ και } \lim_{n\to+\infty}(\frac{3}{2n}) = +\infty\end{array}\right\} \Rightarrow$$

$$\Rightarrow \lim_{n\to+\infty}(\beta_n) = (e)^{\frac{9}{2}}(1) \Rightarrow \boxed{\lim_{n\to+\infty}(\beta_n) = (e)^{\frac{9}{2}}}$$

$$\lim_{n\to+\infty}(\gamma_n) = \lim_{n\to+\infty}\frac{\ln(\sqrt{n+1}) - \ln(\sqrt{n})}{n} = \lim_{n\to+\infty}\frac{\ln\left(\sqrt{\frac{n+1}{n}}\right)}{n} \Rightarrow$$

$$\Rightarrow \lim_{n\to+\infty}(\gamma_n) = \frac{\lim_{n\to+\infty} \ln\left(\sqrt{1+\frac{1}{n}}\right)}{(n)} \Rightarrow$$

$$\Rightarrow \lim_{n\to+\infty}(\gamma_n) = \frac{1}{2}\left(\frac{1}{n}\right)\lim_{n\to+\infty} \ln\left(1+\frac{1}{n}\right) \Rightarrow$$

$$\Rightarrow \lim_{n\to+\infty}(\gamma_n) = \frac{1}{2}\lim_{n\to+\infty} \ln\left(1+\frac{1}{n}\right)^{\frac{1}{n}} = \frac{1}{2}\ln\left[\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{1}{n^2}} \Rightarrow$$

$$\left.\begin{array}{c}\Rightarrow \frac{1}{2}\ln\left[\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{1}{n^2}} \\ \Rightarrow n > 0 \text{ και } \lim_{n\to+\infty}(n) = +\infty \Rightarrow \lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n} = e \\ \lim_{n\to+\infty}(\frac{1}{n^2}) = 0\end{array}\right\} \Rightarrow$$

$$\Rightarrow \lim_{n\to+\infty}(\gamma_n) = \frac{1}{2}\ln(e)^{\lim_{n\to+\infty}\left(\frac{1}{n^2}\right)} = \frac{1}{2}\ln(e)^0 = \frac{1}{2}\ln(1) = 0 \Rightarrow$$

$$\boxed{\Rightarrow \lim_{n\to+\infty}(\gamma_n) = 0}$$

Άσκηση (για επίλυση)

Απάντηση:

$$\lim_{n\to+\infty}(\alpha_n) = \frac{1}{2}$$

Θέμα Εξετάσεων

$$\lim_{n\to+\infty}\left(\frac{n+3}{n+2}\right)^n = \lim_{n\to+\infty}\left(\frac{n+2+1}{n+2}\right)^n = \lim_{n\to+\infty}\left(1+\frac{1}{n+2}\right)^n \Rightarrow$$

$$\Rightarrow \lim_{n\to+\infty}\left(\frac{n+3}{n+2}\right)^n = \left[\lim_{n\to+\infty}\left(1+\frac{1}{n+2}\right)^{n+2-2}\right] \Rightarrow$$

$$\Rightarrow \lim_{n\to+\infty}\left(\frac{n+3}{n+2}\right)^n = \frac{\lim_{n\to+\infty}\left(1+\frac{1}{n+2}\right)^{n+2}}{\lim_{n\to+\infty}\left(1+\frac{1}{n+2}\right)^2} \Rightarrow$$

$$\Rightarrow \boxed{\lim_{n\to+\infty}\left(\frac{n+3}{n+2}\right)^n = \frac{e}{1^2} = e}$$

Θέμα Εξετάσεων

Απάντηση:

$$\lim_{n\to+\infty}(\alpha_n) = 0$$