

HMMY (ΦΕΒΡΟΥΑΡΙΟΣ 2012) - Άσκηση 1

ΛΥΣΗ

$$\alpha) f(x) = \begin{cases} c \cdot 2^{-x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

$$2^{-x} = e^{\ln(2^{-x})} = e^{(-x)\ln 2}$$

$$\begin{aligned} \int_{-\infty}^{+\infty} f(x) dx &= 1 \Rightarrow \int_{-\infty}^0 f(x) dx + \int_0^{+\infty} f(x) dx = 1 \Rightarrow \\ &\Rightarrow \int_{-\infty}^0 0 dx + c \int_0^{+\infty} e^{(-x)\ln 2} dx = 1 \\ &\Rightarrow c \left[\frac{e^{-x\ln 2}}{-\ln 2} \right]_0^{+\infty} = 1 \Rightarrow -\frac{c}{\ln 2} \left[\lim_{x \rightarrow +\infty} e^{(-\ln 2)x} - e^0 \right]_0^{+\infty} = 1 \Rightarrow \\ &\Rightarrow -\frac{c}{\ln 2} \left[\frac{1}{\lim_{x \rightarrow +\infty} e^{x(\ln 2)}} - 1 \right] = (1) \Rightarrow -\frac{c}{\ln 2} [0 - 1] = (1) \Rightarrow \\ &\Rightarrow \frac{c}{\ln 2} = 1 \Rightarrow \boxed{c = \ln 2} \end{aligned}$$

$$\begin{aligned} P[T > 5] &= P[5 < T < +\infty] = \int_5^{+\infty} f(x) dx = \int_5^{+\infty} (\ln 2)(e^{(-\ln 2)(x)}) dx \Rightarrow \\ &\Rightarrow P[T > 5] = (\ln 2) \left[\frac{e^{-x\ln 2}}{(-\ln 2)} \right]_5^{+\infty} \Rightarrow \\ &\Rightarrow P[T > 5] = (2)^{-5} \Rightarrow \boxed{P[T > 5] = (0.03125)} \end{aligned}$$

$$\beta) \left. \begin{matrix} E[T] = \frac{1}{\alpha} \\ \alpha = +\ln 2 \end{matrix} \right\} \Rightarrow \boxed{E[T] = \frac{1}{\ln 2}}$$

$$V[T] = \frac{1}{\alpha^2} \Rightarrow \boxed{V[T] = \left(\frac{1}{\ln 2} \right)^2}$$

γ) Α: “εξυπηρετητής λειτουργικός” $\Rightarrow P(A) = 0.8$

Β: “εξυπηρετητής μη λειτουργικός” $\Rightarrow P(B) = 0.2$

$$\begin{aligned} P[T > 5] &= P(A) P[T > 5 | A] + P(B) P[T > 5 | B] \\ &\left. \begin{matrix} P[T > 5 | A] = 0.03125 \text{ (από ερώτημα } (\alpha) \text{)} \\ P[T > 5 | B] = 1 \end{matrix} \right\} \Rightarrow \\ &\Rightarrow P[T > 5] = (0.8)(0.03125) + (0.2)(1) \Rightarrow \\ &\Rightarrow \boxed{P[T > 5] = (0.225)} \end{aligned}$$

δ)

$$\left. \begin{aligned} P[B | T > 5] &= \frac{P[B \cap T > 5]}{P[T > 5]} \\ P[T > 5 | B] &= \frac{P[T > 5 \cap B]}{P(B)} = \frac{P[B \cap T > 5]}{(0.2)} \end{aligned} \right\} \Rightarrow$$

$$P[B | T > 5] = \frac{(0.2)(1)}{(0.225)} \Rightarrow \boxed{P[B|T > 5] = (0.8888)}$$