

Άσκηση (για επίλυση)

Απάντηση:

$$\lim_{n \rightarrow +\infty} (\alpha_n) = 1$$

Άσκηση

$$\begin{aligned}\lim_{n \rightarrow +\infty} (\alpha_n) &= \lim_{n \rightarrow +\infty} \left(\frac{n^2 - 1}{n^2 + 1} \right)^{n^2} = \lim_{n \rightarrow +\infty} \left(\frac{\frac{n^2}{n^2} - \frac{1}{n^2}}{\frac{n^2}{n^2} + \frac{1}{n^2}} \right)^{n^2} \Rightarrow \\ \Rightarrow \lim_{n \rightarrow +\infty} (\alpha_n) &= \lim_{n \rightarrow +\infty} \left(\frac{1 - \frac{1}{n^2}}{1 + \frac{1}{n^2}} \right)^{n^2} = \frac{\lim_{n \rightarrow +\infty} \left(1 - \frac{1}{n^2} \right)^{n^2}}{\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n^2} \right)^{n^2}} \Rightarrow \\ \Rightarrow \lim_{n \rightarrow +\infty} (\alpha_n) &= \frac{\left[\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{(-n^2)} \right)^{n^2} \right]^{-1}}{\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n^2} \right)^{n^2}} \Rightarrow \\ \Rightarrow \lim_{n \rightarrow +\infty} (\alpha_n) &= \frac{e^{-1}}{e} \Rightarrow \boxed{\lim_{n \rightarrow +\infty} (\alpha_n) = \frac{1}{e^2}}\end{aligned}$$

$$\begin{aligned}\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n^2} \right)^{n^2} &= e, \text{ αφού } n^2 > 0 \text{ και } \lim_{n \rightarrow +\infty} n^2 = +\infty \\ \lim_{n \rightarrow +\infty} \left(1 + \frac{1}{(-n^2)} \right)^{-n^2} &= e, \text{ αφού } (-n^2) < 0 \text{ και } \lim_{n \rightarrow +\infty} (-n^2) = -\infty.\end{aligned}$$

$$\begin{aligned}\lim_{n \rightarrow +\infty} (\beta_n) &= \lim_{n \rightarrow +\infty} \left(\frac{2n + 3}{2n} \right)^{3n+2} = \lim_{n \rightarrow +\infty} \left(1 + \frac{3}{2n} \right)^{3n+2} \Rightarrow \\ \Rightarrow \lim_{n \rightarrow +\infty} (\beta_n) &= \lim_{n \rightarrow +\infty} \left(1 + \frac{3}{2n} \right)^{3n} \lim_{n \rightarrow +\infty} \left(1 + \frac{3}{2n} \right)^2 \Rightarrow\end{aligned}$$

$$\Rightarrow \lim_{n \rightarrow +\infty} (\beta_n) = \left[\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{\frac{2n}{3}} \right)^{\frac{2n}{3}} \lim_{n \rightarrow +\infty} \left(1 + \frac{3}{2n} \right)^2 \right] \Rightarrow$$

$$\Rightarrow \lim_{n \rightarrow +\infty} \left(\frac{3}{2n} \right) = 0$$

$$\left[\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{\frac{2n}{3}} \right)^{\frac{2n}{3}} = e, \text{ αφού } \frac{2n}{3} > 0 \text{ και } \lim_{n \rightarrow +\infty} \left(\frac{3}{2n} \right) = +\infty \right] \Rightarrow$$

$$\Rightarrow \lim_{n \rightarrow +\infty} (\beta_n) = (e)^{\frac{9}{2}}(1) \Rightarrow \boxed{\lim_{n \rightarrow +\infty} (\beta_n) = (e)^{\frac{9}{2}}}$$

$$\lim_{n \rightarrow +\infty} (\gamma_n) = \lim_{n \rightarrow +\infty} \frac{\ln(\sqrt{n+1}) - \ln(\sqrt{n})}{n} = \lim_{n \rightarrow +\infty} \frac{\ln\left(\sqrt{\frac{n+1}{n}}\right)}{n} \Rightarrow$$

$$\Rightarrow \lim_{n \rightarrow +\infty} (\gamma_n) = \frac{\lim_{n \rightarrow +\infty} \ln\left(\sqrt{1 + \frac{1}{n}}\right)}{(n)} \Rightarrow$$

$$\Rightarrow \lim_{n \rightarrow +\infty} (\gamma_n) = \frac{1}{2} \left(\frac{1}{n} \right) \lim_{n \rightarrow +\infty} \ln\left(1 + \frac{1}{n}\right) \Rightarrow$$

$$\Rightarrow \lim_{n \rightarrow +\infty} (\gamma_n) = \frac{1}{2} \lim_{n \rightarrow +\infty} \ln\left(1 + \frac{1}{n}\right)^{\frac{1}{n}} = \frac{1}{2} \ln \left[\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^n \right]^{\frac{1}{n^2}} \Rightarrow$$

$$\Rightarrow \frac{1}{2} \ln \left[\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^n \right]^{\frac{1}{n^2}} \Rightarrow$$

$$\Rightarrow n > 0 \text{ και } \lim_{n \rightarrow +\infty} (n) = +\infty \Rightarrow \lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^n = e \Rightarrow$$

$$\lim_{n \rightarrow +\infty} \left(\frac{1}{n^2} \right) = 0$$

$$\Rightarrow \lim_{n \rightarrow +\infty} (\gamma_n) = \frac{1}{2} \ln(e)^{\lim_{n \rightarrow +\infty} \left(\frac{1}{n^2} \right)} = \frac{1}{2} \ln(e)^0 = \frac{1}{2} \ln(1) = 0 \Rightarrow$$

$$\boxed{\Rightarrow \lim_{n \rightarrow +\infty} (\gamma_n) = 0}$$

Άσκηση (για επίλυση)

Απάντηση:

$$\lim_{n \rightarrow +\infty} (\alpha_n) = \frac{1}{2}$$

Θέμα Εξετάσεων

$$\begin{aligned} \lim_{n \rightarrow +\infty} \left(\frac{n+3}{n+2} \right)^n &= \lim_{n \rightarrow +\infty} \left(\frac{n+2+1}{n+2} \right)^n = \lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n+2} \right)^n \Rightarrow \\ &\Rightarrow \lim_{n \rightarrow +\infty} \left(\frac{n+3}{n+2} \right)^n = \left[\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n+2} \right)^{n+2-2} \right] \Rightarrow \\ &\Rightarrow \lim_{n \rightarrow +\infty} \left(\frac{n+3}{n+2} \right)^n = \frac{\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n+2} \right)^{n+2}}{\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n+2} \right)^2} \Rightarrow \end{aligned}$$

$$\Rightarrow \lim_{n \rightarrow +\infty} \left(\frac{n+3}{n+2} \right)^n = \frac{e}{1^2} = e$$

Θέμα Εξετάσεων

Απάντηση:

$$\lim_{n \rightarrow +\infty} (\alpha_n) = 0$$